Summary

In this dissertation we consider only projective lines over associative rings with unity. The aim of this work is to characterize projective lines over finite rings. Our research is motivated, inter alia, by the papers of Blunck and Havlicek. In the first chapter, we present necessary information about graphs, groups and rings.

In the second chapter, we define the projective line, distant, parallel and adjacency relations on the projective line and we give their properties.

The third chapter is devoted to a problem of the description of a distant graph $G(R, \Delta)$ of the projective line over any finite ring R. We give complete characterization of the distant graph of the projective line over the ring of lower/upper triangular matrices of finite dimension over a division ring. We also describe all isomorphisms and automorphisms preserving distant and parallel relations on the projective line over any semiprimary ring.

The fourth chapter is about distant graphs of the projective lines over finite rings. They can be described via characterizations of distant graphs $G(R, \Delta)$, where R are full matrix rings $M_n(q)$ over fields of order q and also they are indecomposable rings of prime power order. In the first case we give a description of maximal cliques, in particular, we describe all partitions of the set of the vertices of the graph on a sum of vertex-disjoint maximal cliques for matrix ring $M_2(2)$. We classify all graphs of projective lines over indecomposable rings of order p^n for any prime p and any natural number $n \leqslant 5$.

The fifth chapter contains the results concerning free cyclic submodules. We prove that in case of semisimple rings and finite commutative rings all free cyclic submodules make up the projective line. We show that free cyclic submodules of module over the ring of lower triangular 3×3 matrices over a field, fall into five distinct orbits under the action of the general linear group over this ring. Four of these orbits are sets of free cyclic submodules generated by the outliers. The study of rings with non-unimodular free cyclic submodules can be based on indecomposable rings of prime power order. We give the classification of all indecomposable rings of order p^n for any prime p and any natural number $n \leqslant 4$, with free cyclic submodules generated by outliers.